Investigation of the Use of Curvelets for Face Recognition

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Outline

The Problem

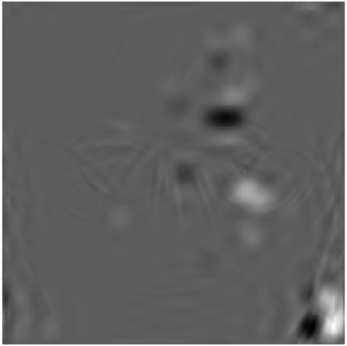


Figure: A principal Eigenface.

For a given picture, decide which face it most closely matches in a database of pictures, and do so quickly and accurately

Defining ‘Face’

In order to have a well defined problem, we must ask,

"What defines a ‘face’?"

The Sobel operators

$$\mathbf{G}_x = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} * \begin{bmatrix} -1 & +1 \\ -1 & +1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix} \quad \text{and}$$

$$\mathbf{G}_y = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} * \begin{bmatrix} +1 & +1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Correlation and the 2d Inner Product

- Correlation:
  $f(x, y) \star h(x, y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f * (m, n) h(x - m, y - n)$
- Vector Operations: $\vec{v} = \mathbb{R}^{m*n} \mapsto \mathbb{R}(m * n) * 1$
  Then, to check the 'closeness' to another face ($\vec{u}$) that has been vectorized, can we use $\vec{u} \cdot \vec{v}$ or at least some sort of comparsion of $\|\vec{v}\|$ and $\|\vec{u}\|$?
- Matrix Multiplication: Matrix multiplication, is just a number of dot products so can we find an appropriate matrix for the projection, whose norm we could take to give us a scalar?

Dimension Reduction

Our Problem:
For a database of images, we have a lot of data. Each image has $m \times n$ pixels which can each have uncorrelated values. The idea behind dimension reduction is that you can preserve the important information, while doing operations that reduce the amount of independent data sources.
A type of dimension reduction we regularly encounter can be acheieved by simple matrix multiplication when the matrix is not full rank. Applying a rank 2 matrix to a $[3, 1]$ vector $\vec{x}$ will result in a matrix whose columns span 2 dimensions. The following example reduces a 3d vector to a line.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \vec{x} = [x_1, x_2, x_3], \; A\vec{x} = \begin{bmatrix} 2x_1 + 4x_2 \\ x_1 + 2x_2 \\ 0 \end{bmatrix}$$

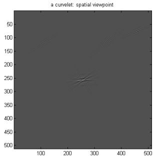
2d Image Bases

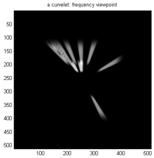
- Quadtree/Bitplane
- Matrix Factorizations
- Fourier Basis
- Wavelet/ X-let
- SVD/2dSVD
- Curvelet

These transforms all have inherent structure to them.

Curvelets

- Localized in **both** time and frequency.
- A Curvelet is indexed by $\gamma_{a,b,\theta} = \gamma_{a,(0,0)}\left(R_\theta(x-b)\right)$ where a is the scale parameter, b is the translation parameter where $b \in \mathbb{R}^2$, and R_θ denotes rotation.
- Tight Frame/ Robust over multiple scales.
- Strong edge detection. Optimal encoding of C^2 edges.
- Parabolic scaling. Self-adjoint. Wave propogation properties.
- Developed originally by UW professor Hart Smith. Further developed by professor David Donoho.
- The CurveLab software I use is available at `http://www.curvelet.org`





Curvelet Picture

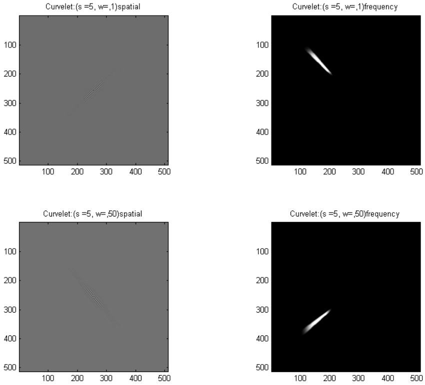


Figure: A few example Curvelets in time and space.

Singular Value Decomposition (SVD)

- The SVD produces a basis that best approximates a given set of data.
- Since images are 2d, to take the SVD, we reshape the vectors of our images, insert them into a matrix as columns, and then operate.
- The form of the SVD is: $X = U\Sigma V^*$, where Σ is diagonal and stores singular values and U and V^* are orthogonal bases.
- Derived from the eigenvalues of covariance matrices XX^* and X^*X.
- Used in EigenFaces. These are just the principal modes (singular values are arranged from greatest to smallest).

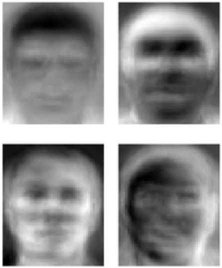


Figure: Example of Eigenfaces taken from Wikipedia.

Covariance

Covariance is key to discussing PCA, LDA, and SVD.

- Given measurements: $A = \{a_1, a_2, ..., a_n$ and $B = \{b_1, b_2, ..., b_n$
- Define variance as: $\sigma_A^2 = \langle a_i a_i \rangle_i$
- Define covariance as: $\sigma_{AB}^2 = \langle a_i b_i \rangle_i$
- Since we want the covariance over all the measures, we can insert our vectors into a matrix and express it that way.
- That gives: $\sigma_{AB}^2 = \frac{1}{n-1} ab^T = s_x = \frac{1}{n-1} XX^T$

This presentation taken from Jon Shlens' 'A Tutorial on Principal Component Analysis'

Decision Making

- K-means
- LDA
- FisherFaces (Fisher Discriminant)
- SVM
- PCA
- Neural Networks

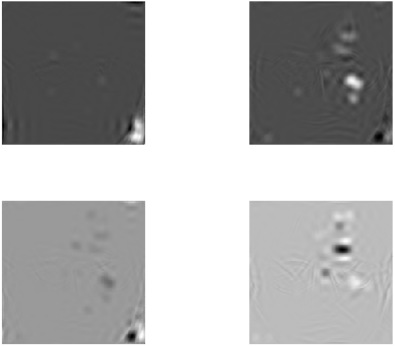
Curvelet Processing

- Georgia Tech Face database (available at `http://www.anefian.com/research/face_reco.htm`)
- crop the images in a way that background information is minimized, equalize their histograms, and perform the curvelet transform on them.
- I am investigating different ways of thresholding the coefficients
- Everything is dependent on completion of my LDA methods.
- Individual Decomposition and re-inversion after thresholding is around 1 sec per image.



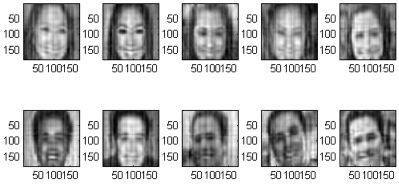
EigenFaces

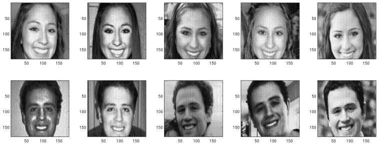
Using the standard SVD, I generated the principle EigenFaces. As I do not yet have my classifier algorithm completed, I cannot tell how useful or accurate they are. At this point, I am balancing thresholding as many coefficients as possible while not losing too much data.



Implementation of 2DSVD

From Chris Ding's paper, I implemented 2DSVD because he made some strong claims about its efficacy. It allows for a low-rank approximation by using less eigenvalues for reconstruction. There is substantial degradation, even when looking at the first few principal modes.





Conclusions

- Curvelets might not be the appropriate tool for fast verification.
- With a carefully created feature vector heuristic, they can be an incredibly powerful tool.
- If we were dealing with noisy images, they would certainly be the tool of choice.
- More thresholding techniques need to be investigated
- Careful statistic analysis of each of the constituent matrices.
- Provide better documentation so others can work quicker.

References

Curvelets

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Statistical Methods

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- ‘2D-LDA: A Statistical Linear Discriminant Analysis for Image Matrix’. Ming Li, Baozong Yuan, Pattern Recognition Letters 26 (2005) 527-532.
- ‘Eigenfaces for Recognition’. M. Turk, A. Pentland. Journal of Cognitive Neurosicence, Vol. 3, No. 1, Win. 1991, pp. 71-86

Thanks!

Download my presentation at
https://s3.amazonaws.com/andy_hw/curvelets.pdf.
Feel free to email me with any questions at: andyasb@uw.edu

Lena: Original

Lena: Curvelet Representation